Pouchet, G., "Des changements de coloration sous l'enfluence des nerfs," *Journ. Anat. Physiol.*, **12**, 1–90, 113–165 (1876).

Taylor, H. F., "Pearl Essence: Its History, Chemistry and Technology," *Rep. Comm. Fish.*, Washington, **1925**, 15–36 (1926).

THE CAUCHY-GOURSAT THEOREM FOR RECTIFIABLE JORDAN CURVES

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In a recent paper[1] Kamke stated that the property expressed by Cauchy's integral theorem had never been proved for the case of a function analytic interior to an arbitrary rectifiable Jordan curve, continuous in the corresponding closed region. A proof was then supplied by Denjoy.[2] The following proof is much more immediate than that of Denjoy, although not so elementary.

THEOREM I. *If C is a rectifiable Jordan curve and if the function $f(z)$ is analytic interior to C, continuous in the corresponding closed region, then we have*

$$\int_C f(z)\,dz = 0.$$

The integral of an arbitrary polynomial $p(z)$ over C is zero, for that integral can be expressed as the limit as n becomes infinite of the integral of $p(z)$ over a suitably chosen closed polygon π_n whose vertices lie on C; the latter integral is clearly zero. The function $f(z)$ of Theorem I, being analytic interior to C and continuous on and within C, can be represented *in the closed interior of* C as the limit of a uniformly convergent sequence of polynomials.[3] This sequence can be integrated over C term by term, so Theorem I is established.

Theorem I extends easily to the case of a limited region D bounded by a finite number of non-intersecting rectifiable Jordan curves, if $f(z)$ is analytic interior to D, continuous in the corresponding closed region. In such a closed region the function $f(z)$ can be expressed as the limit of a uniformly convergent sequence of rational functions of z whose poles lie exterior to the closed region.[4] The integral of such a rational function over the complete boundary of D is zero; hence the corresponding integral of $f(z)$ is also zero.

In particular, Cauchy's integral *formula* is valid under the hypothesis of Theorem I, or under the more general hypothesis just mentioned.

[1] *Math. Zeit.*, **35**, 539–543 (1932).
[2] *Paris Compt. Rend.*, **196**, 29–33 (1933).
[3] Walsh, *Math. Annal.*, **96**, 430–436 (1926).
[4] Walsh, *Ibid.*, 437–450 (1926).

AN INEQUALITY FOR LEGENDRE SERIES COEFFICIENTS

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Let the set $\{\varphi_n(x)\}$ be an ortho-normal set of functions on the interval (a, b) and let M be a constant such that

$$|\varphi_n(x)| \leqq M, \quad n = 0, 1, 2, \ldots;$$

then the Fourier expansion of any integrable function $f(x)$ will be

$$f(x) \sim \sum_0^\infty c_n \varphi_n(x),$$

where

$$c_n = \int_a^b f(t)\varphi_n(t)dt.$$

We introduce the notation

$$J_p(f) = J_p = \left(\int_a^b |f(x)|^p dx\right)^{\frac{1}{p}}, \quad S_{p'}(f) = S_{p'} = \sum_0^\infty |c_n|^{p'},$$

where $1 < p \leqq 2$, $p' \geqq 2$, $\dfrac{1}{p} + \dfrac{1}{p'} = 1$. It will be assumed throughout that p and p' satisfy these relations.

In terms of this notation F. Riesz's[1] theorems can be stated, for a uniformly bounded ortho-normal set of functions, as follows.

(A) If $f(x)cL_p$, then

$$S_{p'} \leqq M^{\frac{2-p}{p}} J_p.$$

(B) If the series $\sum_0^\infty |c_n|^p$ is convergent, then the constants c_n are the Fourier coefficients of a function $f(x)cL_{p'}$, and, moreover,

$$J_{p'} \leqq M^{\frac{2-p}{p}} S_p.$$

As was called to my attention by Professors Hille and Tamarkin, in the case of the expansion of the function

$$f(x) = \left(\frac{2}{1-x}\right)^\alpha$$